

1.1 Simple Interest

Friday, September 06, 2013
9:30 AM

Ex If you invest \$100 in a simple interest bank account on your birthday, that earns 5% (per year), how much will you have on your b'day the following year?

$$\text{interest: } 0.05 \times \$100 = \$5$$

$$\text{total \$: } \$100 + \$5 = \$105$$

$$(\text{another way: } 1.05 \times \$100 = \$105)$$

- this was a "fixed interest rate" (guaranteed not to change)
- the \$100 was the "principal" (initial investment)
- one year was the "term" (contracted length that you invested for)
- the investment "matured" (ended) on your b'day

Ex Now assume that you let the investment remain invested for 10 years (term), instead of 1. How much interest would you earn?

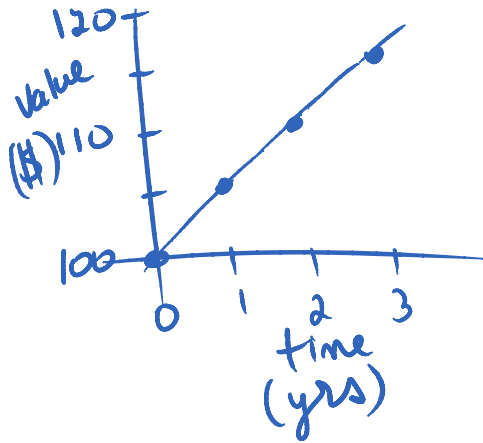
year	int	total
	\$5 per year	\$50

$$\$5 \text{ per year} \times 10 \text{ years} = \$50$$

year	int	total
0	—	100
1	5	105
2	5	110
⋮	⋮	⋮

$$\$5 \text{ per year} \times 10 \text{ years} = \$50$$

- this is "simple interest": you only get interest on the original \$100 (no interest on the interest - that would be compound, next section)



Formula

$$\star A = P + \overbrace{Prt}^{\text{Interest}} \leftarrow \begin{array}{l} \text{\# of} \\ \text{years} \\ \text{(term)} \end{array}$$

\uparrow future value \uparrow Principal \uparrow interest rate (decimal form)

$$\text{or } A = P(1 + rt)$$

$$\star I = Prt \leftarrow \text{calculate the interest earned}$$

$$\star \text{Rate of Return} = \frac{\$ \text{ earned (interest)}}{\$ \text{ invested (principal)}}$$

1.2 + 1.3 Compound Interest

Monday, September 09, 2015
9:23 AM

Compound interest - interest that is paid on the principal plus any accumulated interest (interest on the interest)

Ex \$1000 @ 3.6% simple interest for 5 years

$$\begin{aligned} I &= Prt \\ &= 1000 \times 0.036 \times 5 = \$180 \quad (\$36/\text{year}) \\ A &\longrightarrow 1000 + 180 = \$1180 \end{aligned}$$

Compared with

\$1000 @ 3.6% compound interest for 5 years

$$\begin{aligned} \text{end of 1st year:} & \quad 1000 \times 0.036 \rightarrow 1000 + 36 = \$1036 \\ \text{end of 2nd year:} & \quad 1036 \times 1.036 = \$1073.296 \\ \text{" " 3rd year:} & \quad 1073.296 \times 1.036 = 1111.935 \\ \text{" " 4th " :} & \quad 1111.935 \times 1.036 = 1151.964 \\ \text{" " 5th " :} & \quad 1151.964 \times 1.036 = \boxed{\$1193.43} \end{aligned}$$

Formula

a simpler way to calculate compound interest:

$$A = P \left(1 + \frac{i}{n} \right)^{tn}$$

Future value \nearrow A

Principal \nearrow P

i \nwarrow given annual interest rate (decimal form)

n \nwarrow $n = \#$ of compounding periods per year

t \nwarrow $t = \#$ of years

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example

$$P = 1000$$

$$i = 0.036$$

$$n = 1 \text{ (assume annually)}$$

$$t = 5$$

$$A = ?$$

ex compounding annually $n=1$
semiannually $n=2$
quarterly $n=4$
monthly $n=12$
weekly $n=52$
daily $n=365$

$$\begin{aligned} A &= P \left(1 + \frac{i}{n} \right)^{tn} \checkmark \\ &= 1000 \left(1 + \frac{0.036}{1} \right)^{5(1)} \checkmark \\ &= 1000 (1.036)^5 \\ &= 1000 (1.193435) \\ &= \$1193.44 \checkmark \end{aligned}$$

$$\text{Interest Earned} = A - P$$

$$= 1193.44 - 1000$$

$$= \$193.44$$

Rule of 72

- estimates the doubling time of an investment
- works for compound interest

- works best for compounding annually

Formula

$$\frac{72}{r}$$

r ← annual interest rate
as a %

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$$\frac{72}{3.6\%} = 20 \text{ years to double the initial principal}$$

Practice

pg 30 # 2, 4ac, 5, 7, 9, 13

Quiz in 4 days